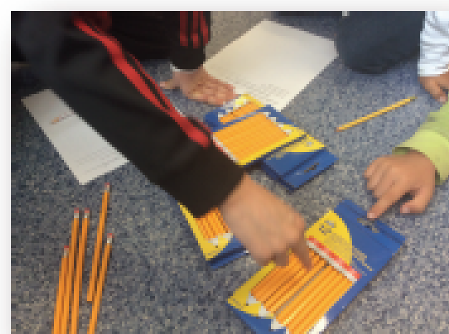
What next?

1) If a student is having difficulty counting, reduce the quantity of objects and ensure that the student is moving the objects as they count. When students first learn to count they often think of quantities as a series of one, and another one and another one. They label each object rather than determining how many altogether. Check to see if the students can count objects with ease and accuracy and whether they know one more or one less without having to count each item. Knowing without counting is a key idea when working with addition and subtraction.

2) If a student is fluent and confident when counting, you may consider providing larger quantities for counting. Another suggestion would be to provide packages of objects and some loose parts (i.e. 7 packages of 12 pencils and 6 loose pencils) and ask the students: "How many pencils? How do you know?"



References

Counting Collections by Julie Kern Schwerdtfeger and Angela Chan, Teaching Children Mathematics, March 2007, pages 356 – 361.

Number Sense Routines: Building Numerical Literacy Every Day in Grades K-3 by Jessica Shumway

Children's Literature

How Many Snails - *Paul Giganti Jr.*

Counting Sheep - *Kathryn Cave*

How Many Seeds in a Pumpkin? – *Margaret McNamara*

One is a Snail, Ten is a Crab - *April Pulley Sayre & Jeff Sayre*

How Many Jelly Beans? – *Andrea Menotti*

created for the BCAMT Reggio-Inspired Mathematics Inquiry Project/SB May 2015/revised August 2015

Counting

What is it?

Counting is more than repeating a rote sequence and recognizing the numerals. Learning to count while simultaneously developing a sense of quantities and number relationships is an important foundation for students. It is essential that the student understands a quantity represents the number and how numbers are related. Counting is finding out how many.

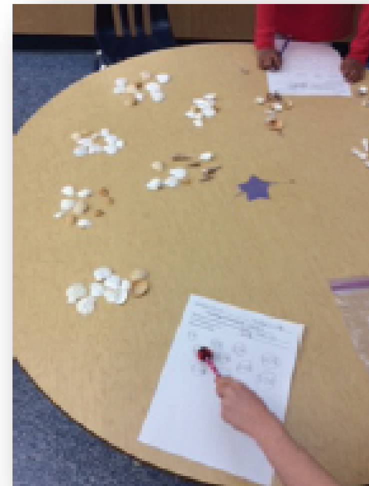
Students need to develop the following skills through multiple counting opportunities:

- correct sequence of number names
- one-to-one correspondence: Saying one number name for each object counted.
- cardinality: The last number said is the quantity counted.
- stability: The quantity of a group does not change if the objects are rearranged
- relative size: Which is more than/less than?
- make connections between number names, quantities and symbols
- counting forwards, backwards and from any starting point
- base-ten structure: How do these numbers go together? How can I count by tens and ones to find out how many?

Students need to develop an understanding of number and counting by:

- Counting forward and backwards (to 10, starting at any given number, to 100, then beyond 100)
- Counting on and back by 1, 2, 10, through decades (19-20, 40-50)
- Skip counting forward and backwards (cornerstone for multiplicative understanding)
- Identifying patterns in our place value system

After a mini lesson, invite students to investigate materials with the focus of thinking about counting the quantities. By providing baskets of objects, wooden numerals, wooden ten frames, and black felt mats, you are encouraging students to explore ways to count and strengthening their ability to connect the count the quantities. Provide a provocation such as; "How many are there? How many do you estimate? or How might you count them?" As the students are investigating, circulate, listen, watch and document the learning. Ask 'nudging' questions as needed.



Gather the students together to share their thinking and strategies. Record the various strategies used. Discuss the 'uncovering' of the math concepts.

What to look for?

Provide the students with a variety of collections (i.e. shells, rocks, twigs)

- Are students able to keep track of the objects when counting?
- Do the students get the same answer when they recount?
- Are students able to accurately count the collection?
- Can the student reasonably estimate the quantity?
- Can the students count forwards, backwards and from any starting point?
- Do the students identify patterns?
- Can the students count on?
- Can the students skip count?
- Are the students able to record the count pictorially and then symbolically?
- When counting, do the students cross decades without hesitation?
- Do the students recognize relationships between numbers?

Why is it important?

Counting has proven to be important to lay the foundation for understanding of the base-ten system, operating on numbers and problem solving. When students develop competence, they not only count with accuracy and ease, but will also develop the sense of the quantity of numbers they are working with. The focus moves from knowing the number they landed on to making reasonable estimates and noting the reasonableness of the outcome of the counting. Research studies have determined that counting experience is fundamental to the construction of number sense and concepts (Baroody 1987, Sinclair and Sinclair 1986).

What to think about?

- Gather a variety of items to count.
- The size of the collection should vary. Begin with collections that might range from 25 to over 200.
- Have cups, bowls, and other containers available for grouping the items.
- Have ten frames and 100 charts available for the students to use.
- Consider how to spark provocations. How many…?
- What questions might you ask to 'nudge' thinking?
- How are you going to celebrate the various counting strategies?

What to do?

Begin with a mini-lesson using a small collection of objects. Ask the students how they might count this collection. Choose a suggestion from the students and count the collection together. Provide access to cups, bowls, mats and ten frames for the students to use. Ask students; "What is important to remember when counting collections?" Students might identify the importance of keeping track of the objects, organizing them into groups, counting forwards/backwards, counting on etc. Ask students; "How might you record the count?"

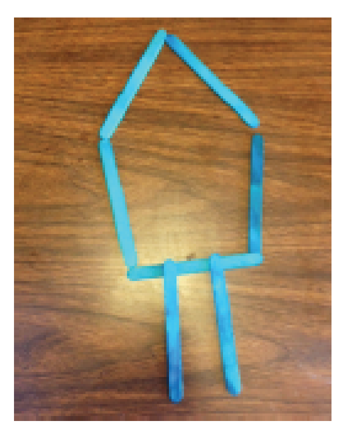
- Does the student demonstrate that there many, yet a finite, number of ways to decompose a quantity?
- Is the student able to work abstractly with a number to decompose versus a quantity of materials?

What next?
1) If a student is having difficulty decomposing, reduce the quantity of objects the student is using. Students may also benefit from having representations of their decomposing to refer to. So, for example if the student is working with 4 and they make 3 and 1, then have the student leave that representation with blocks and then take another 4 blocks and decompose 4 in a different way. This encourages the student to check and see if they have or have not already decomposed the number in that way. Prompt the student with questions like, can you decompose that into three parts? four parts? etc to help her with different ways of decomposing.

2) If a student is fluent and confident with decomposing, move towards using number symbols and larger numbers. A challenging question, involving mathematical competencies of justification and proof, is "*How do you know you have found all the ways to decompose this number?*"

References

Number Sense Routines *by Jessica Shumway*
Common Core Math for Parents *by Christopher Danielson*
How Children Learn Number Concepts *by Kathy Richardson*
Developing Essential Understanding of Number & Numeration: Pre-K - Grade 2 *by National Council of Teachers of Mathematics (NCTM)*
Complex Counting in Kindergarten *by Eve Sci, Kirten Sendrowski Kircher and Heather Shook, Teaching Children Mathematics*, March 2016



Children's Literature
12 Ways to Make 11 by *Eve Merriam*
Ten Black Dots *by Donald Crews*
1 Cookie, 2 Chairs, 3 Pears *by Jane Brocket*
One is a Snail, Ten is a Crab *by A. Pulley Sayre*

Decomposing Numbers

What is it?
Decomposing and composing quantities or numbers are related concepts. Decomposing is essentially "breaking" a quantity into parts, such as ten can be decomposed into five and four and one. Alternately, a quantity of ten can be composed of parts put together to make ten, such as four and four and two.
It is important to think about decomposing numbers into more than two parts (as often seen in "number bond" representations) so that students have a broader understanding of quantities and fluency with decomposing numbers into multiple parts.

Unlike partitioning, which is usually considered a fractional concept used when "cutting" shapes into parts, and involves equi-partitioning or equal parts (Danielson, 2015), decomposing and composing quantities do not need to be of equal parts.

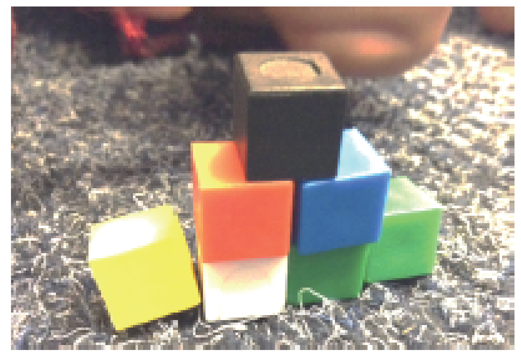
"Composing and decomposing small sets of objects can allow young students to see the whole groupas well as the subsets that created the whole." ~Sci et al, 2016

Why is it important?
Being fluent with decomposing and composing numbers develops a flexibility in understanding and calculating with numbers. In Kindergarten and Grade 1, it is essential that children develop a strong sense of both five and ten and the relationship between these quantities. The use of five and ten frames can support this understanding. Decomposing and composing numbers is a critical component of number sense.

- *Note that students can also make a math-to-math connection as they think about decomposing and composing shapes, for example, a rectangle can be decomposed into two triangles.*

Materials to support decomposing understanding: loose parts, Unifix cubes, five and ten frames, blank hundred grids, rekenreks, abacuses, digit cards, wooden numbers, dominoes, dice, divided containers or parts-whole mats.

What to do?



Gather the students together and provide them with a variety of materials such as glass gems, cubes or rocks. Begin with an opening conversation about decomposing by asking, "How many ways can you make 5?" Have the students use materials to show various ways to decompose the quantity of 5 and share their thinking. Record the various combinations with drawings and/or by saying '3 and 2 is 5' or '3 +2=5' depending on the understanding of the students. After a mini lesson, invite the students to investigate materials with a focus of representing quantities by decomposing in different ways. By providing a basket of objects, numerals, wooden ten frames, and black felt mats, you are encouraging student to explore ways to decompose and compose quantities. Provide provocations such as; *"How many ways can you make…? What parts and whole relationships are represented on dominoes? What do you notice when you decompose numbers?"*

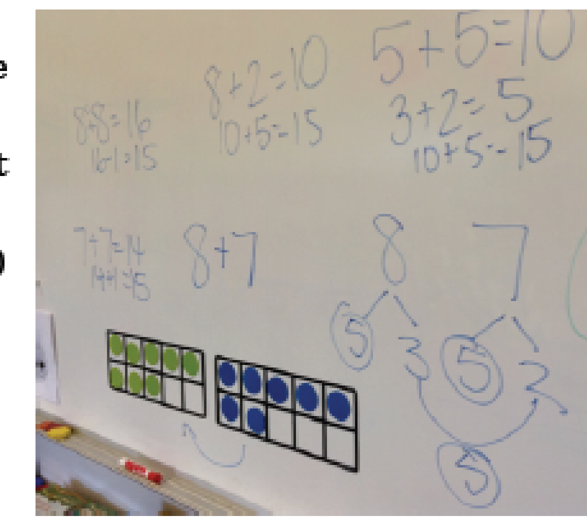
What to look for?

As a student works with a quantity to decompose:

- Is the student able to keep track of the quantity when counting?
- Is the student able to accurately count the quantity/set?
- Can the student reasonably estimate the quantity?
- Is the student able to subitize the quantity? Explain how she can "see" it?
- Does the student move the materials to create different parts?
- Does the student consider a pattern in how they decompose – two parts, three parts, four parts, etc?
- How does the student keep track of the different ways they have decomposed the quantity?

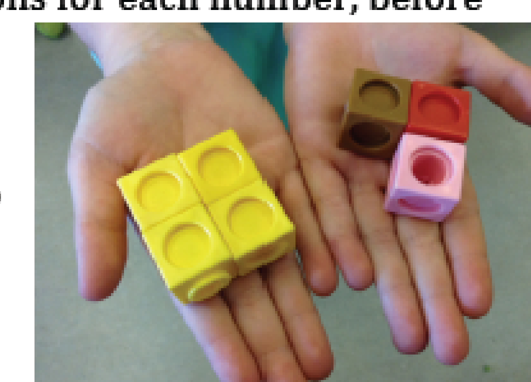
"As children learn the combinations that make up numbers to 10, they will reach the point where they know the parts so well, they can identify the missing parts when they know the total and one part." ~Kathy Richardson

Students need a strong understanding of parts-whole relationships in order to use mental mathematics strategies to add, subtract, multiply and divide. This fluency is often seen during the routines of "Number Talks" when a question is presented to a class to solve and then the students share different approaches and strategies. Students who are able to fluently decompose quantities and then compose quantities are able to adapt their calculations to different sets of numbers. For example, when adding 8+7, a student might notice that 8 is just two away from 10 so she decomposes 7 into 2 and 5 and uses the 2 to add to 8 to make a 10. She then is able to efficiently add the 10 to the remaining part (5) to make 15. In another number talk, the same student may notice that to solve 47 + 26 that 47 is close to 50 so decomposes 26 into 23 and 3 and uses the 3 to add to 47 to make 50 and then adds on the remaining 23 to total 73. Decomposing larger numbers by place value is also an important concept and skill that students can use when adding, subtracting, multiplying and dividing.



What to think about?

- Before students are able to decompose numbers they need to recognize that smaller numbers are contained in larger numbers (i.e. the quantity of 7 is not just a collection of seven 1s, but is composed of 3 and 3 and 1 more, for example).
- Students need lots of practice decomposing and composing quantities or numbers, working with various combinations for each number, before they will be able to decompose numbers to find the 'missing part.'
- Experience with composing and decomposing numbers will help to develop computational fluency.



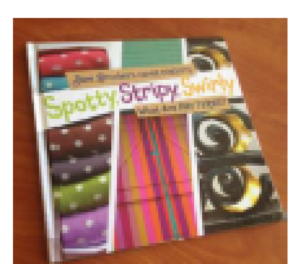
part that repeats over and over. With concrete representations have students use their hands to frame the core of the pattern.
1) If a student is having difficulty creating patterns other then a typical AB pattern in a line, focused only the attribute of colour, intentionally provide three colours of material and ask student to create a pattern using all three colours, providing support as necessary. Also try providing different materials (ie. all yellow cubes) to help the student consider position or orientation or provide different mats such as spiral, circle or grid to help the students think about patterns in different ways.

What next?
2) If a student is fluent and confident creating and describing patterns, provide opportunities to investigate different type of geometric patterns such as mosaics and mandalas or patterns in other areas of interest such as sports or music. Extend students thinking about patterns by asking them to make connections to and seek numerical relationships within the patterns.

References
Big Ideas from Dr. Small: Grades K-3, Chapter 1-Patterns and Algebra Marian Small
Making Math Meaningful to Canadian Students, K-8 by Marian Small
Developing Number Concepts: Counting, Comparing and Pattern by Kathy Richardson
Is it a Pattern? by Lynn M. McGarvey, Teaching Children Mathematics, May 2013, Vol. 19, Issue 9, pages 564-571.
Sorting and Patterning in Kindergarten: From Activities and to Assessment by Elizabeth J. Ziemba and Jo Hoffman, Teaching Children Mathematics, January 2006, pages 236-241.

Real World Connections
Patterns are evident in a child's everyday surroundings – in the tiling of a floor, artistic representations and the number patterns found within street addresses. There is a predictable nature to patterns that transfers to many real world applications such as music, weather and language.

Children's Literature
Spotty, Stripy, Swirly: What are Patterns? by Jane Brocket
Beep Beep, Vroom Vroom! by Stuart J. Murphy
Sorting through Spring by Lizann Flatt
Pattern Fish and Pattern Bugs by Trudy Harris



created for the BCAMT Reggio-Inspired Mathematics Inquiry Project/SB&JN October 2015

Patterning

What is it?
Patterning involves being able to recognize, describe, create and extend patterns that have an identified regularity. Patterning involves big mathematical ideas of order and regularity. Students are able to generalize and explain a pattern "rule" for concrete and pictorial patterns.

- Repeating patterns: In repeating patterns, the identified regularity is a "core" or the part of the pattern that repeats over and over. For example, a student might create a long linear pattern using coloured gems alternating blue and green. The child identifies that it is the blue and green part that repeats.

- Increasing and decreasing patterns: In increasing and decreasing patterns, the identified regularity is the constant amount that affects each term in a predictable way. For example, a student creates a staircase pattern using wooden cubes explaining that each stair increase by two cubes.

Why is it important?
Patterning is often considered the foundation of mathematical thinking. Mathematics is often referred to as the science of patterns (Devlin, 2001). The ability to see, describe, compare and generalize patterns is essential for mathematical thinking in algebra and other mathematical topics. Patterning is important because it allows students to seek connections and see relationships, particularly visual-spatial and additive or multiplicative relationships.

What to think about?

- Begin by providing students with a collection of materials to sort and describe (eg. buttons or coloured cubes). Students need to be able to pay attention to and describe the attributes of the materials such as colour, size, postion, etc.
- Provide students with examples of multiple concrete and visual patterns, ranging from simple and complex. During class discussions or small group work, elicit descriptions of patterns, using mathematical vocabulary to support students' thinking. For example, terms like "core", "term", "label", "attribute", "repeat", "increase" and "decrease" should be infused into the math talk in the classroom.
- A variety of both mathematically structured materials such as Unifix cubes and patterns blocks as well as other materials such as rocks, shells and buttons offer different possibilities for patterning and mathematical thinking.
- Look for opportunities when students are working with materials to introduce different ways to describe and label patterns such as using letter coding (ABABAB) or seeking numerical relationships.

What to do?

Gather students together to share a concrete model of a pattern or a photograph of a pattern found in the "real world" and ask students: *What do you notice?* Listen for the language the students are using and weave in mathematical vocabulary as applicable.

Ask students to consider: *What is a pattern? What makes a pattern a pattern?*

After this opening, students could move to investigating materials with the focus on thinking about patterns with the prompt: *What can you find out about patterns?* Different types of materials could be presented on different tables, including both mathematically structured materials and other natural or found materials. By adding numeral or letter cards, an opportunity for labeling patterns is provided. Likewise, a collection of mats such as spirals or open grids could be presented in a provocation for students to continue their exploration of patterning.

Some examples of patterning provocations include:

What different patterns can you make?
What patterns are inspired by these materials?
Can patterns be repeating and increasing at the same time?
What happens when circles and patterns meet?
Where do you see patterns in your world?

What to look for?

Provide students with a small collection of materials such as buttons or coloured cubes.

> *Are students able to sort and describe the attributes of the materials? Do they see how the materials are the same and how they are different?*

Prompt students to create a pattern with the materials.

> *What kind of patterns do students create? Do they only create the same pattern (ie. AB) but with different materials? Are students able to identify the core of the pattern? Are students able to extend the pattern in either direction/at either end?*

After an extended time working with the materials and patterns, look for developing understanding of the big ideas in patterning.

> *Are students creating patterns with increasing complexity over time? Are they able to compare patterns – how are these two patterns the same and different? How could you change this pattern to make a different pattern?*

What next?

1) If a student is having difficulty identifying the core of a repeating pattern (eg. AB or ABBC) or the constant change of an increasing or decreasing pattern (eg. add 3 or doubling), increase the focus of this aspect in class or small group discussions. For example, with linear repeating patterns drawn out on a chart, have students identify and come up to draw a box around the

What next?

2) If a student is fluent and confident when subitizing you may consider using visual referents for estimating larger quantities, if the student has an understanding of greater numbers. For example, show a dot card of 5 and have the student quickly identify that quantity. Then look at a large quantity of the same-sized dots (ie 20) and have them use the 5 as a visual referent to help estimate the larger quantity. Students that are fluent subitizers can also begin to use different visual patterns beyond dots and ten frames such as twenty frames and open hundred grids. Confident subitizers can also begin to apply this skill to mental math strategies by being able to visualize quantities in ten frames and combining the dots or filled spaces to add and subtract.

References

Subitizing: What is it? Why teach it? by Douglas Clements, Teaching Children Mathematics, March 1999, pages 400-405.
Number Sense Routines: Building Numerical Literacy Every Day in Grades K-3 by Jessica Shumway
Number Talks by Sherry Parrish

Real World Connections

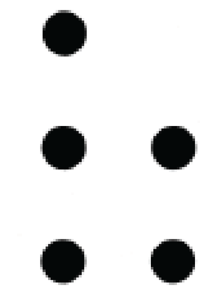
We use subitizing in many ways – to help us "chunk" and remember phone numbers, digit grouping of large numbers, recognize numbers on cards, dice, etc during games and to help us visualize and estimate quantities or measurements.

Children's Literature

Ten Black Dots by Donald Crews
The Cheerios Counting Book by Barbara Barbieri McGrath

created for the BCAMT Reggio-Inspired Mathematics Inquiry Project/JN May 2015/revised August 2015

Subitizing



What is it?

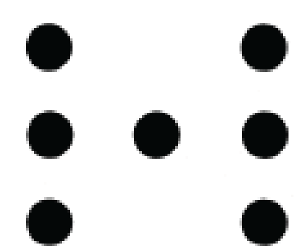
Subitizing is the ability to see a small quantity of objects and know at a glance how many. There are two types of subitizing – perceptual and conceptual.

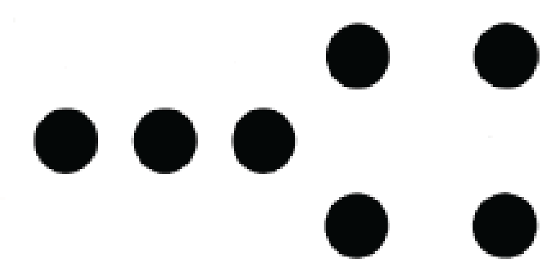
- Perceptual subitizing is recognizing the quantity of a small group of objects without using other mathematical processes.

- Conceptual subitizing involves a more advanced organizational role and involves being able to visualize patterns, decompose and see a quantity in parts. For example, when seeing a dot pattern of six on a die, it may be seen as three and three. Although we tend to focus on spatial patterns, subitizing can also be used with temporal and kinesthetic patterns such as finger, rhythmic and auditory patterns.

Why is it important?

Subitizing has proven to be a key indicator of students' mathematical development and thus should be taught, experienced and practiced in primary classrooms. There are direct links to students' future number system knowledge and understanding as well as computational fluency. Research studies have determined that being able to subitize quantities up to and including 4 by age 5 is a significant milestone. (Desoete & Grgoire, 2006; Fischer, Gebhardt and Hartness, 2008; Landerl, Bevan & Butterworth, 2004; Nichols, 2006; Yun et al, nd).





What to think about?

- Begin with subitizing quantities of 2, 3 and 4 with students.
- Children usually find rectangular arrangements (arrays) the easiest to subitize, followed by linear, circular and scrambled arrangements.
- Arrangements should not be embedded in a pictorial context, simple shape forms should be used, regular arrangements usually with symmetry should be emphasized, and the arrangements should have good figure-ground contrast.

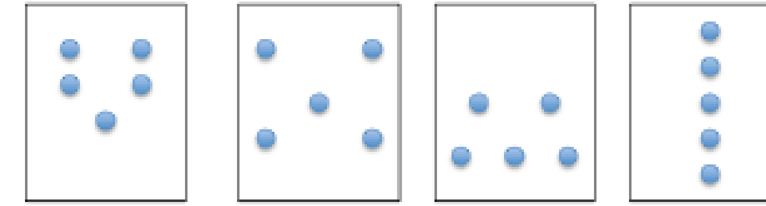
What to do?

Begin with an opening conversation by holding up dot cards for a few seconds and ask students to call out what how many dots they see. As quantities increase, have students explain how they saw the dots, for example if they broke a larger quantity into two parts (conceptual subitizing). The students can also have a set of counters in front of them and when a dot card is held up, they visualize the amount and then build the set with the counters. Ten frames can also be used like this in a mini-lesson.

Quick Images/Flash and Say
Flash and Describe
Flash, Visualize and Build

After this opening, students could move to investigating materials with the focus on thinking about numbers. By presenting a collection of dot cards alongside a basket of glass gems on a mat, you are inviting students to re-create the dot patterns, strengthening their understanding of visual patterns. By adding dotted dice or dominoes, a provocation is suggesting interacting with the materials in game play while practicing subitizing. Likewise, a collection of ten frame cards, mats or open frames could be presented in a provocation for students to continue their exploration of subitizing and visual patterns.

Different arrangements lead to different ways of visually decomposing that quantity:



What to look for?

Begin with very small quantities of 2, 3 and 4 in different arrangements.

Are students instantly able to recognize the amount as a whole or are they visually counting the dots?

The human brain can generally perceptually subitize a quantity up to and including seven items. This develops over time and you may see a range of quantity that your students can subitize. Start where your students are and build from there.

To what quantity are your students able to subitize?

Does the arrangement of the visual pattern affect the quantity they can subitize?

When moving to quantities of 5, 6 and 7 look for whether students are using perceptual or conceptual subitizing. Ask: *"How did you see those dots?" "Could you see those dots in a different way?"*

How are your students subitizing?

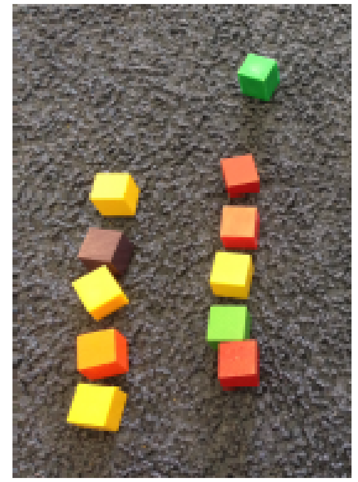
Can they visualize the dot patterns in more than one way?

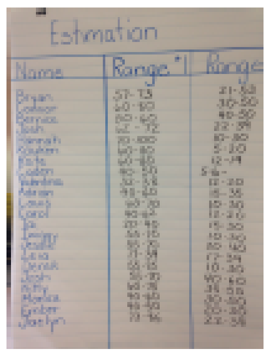
What next?

1) If a student is having difficulty subitizing, reduce the quantity of dots and ensure that the dots are darkly coloured against a light background and that the dots are not too close together. Place a dot pattern card in front of a child and have him or her look at it carefully, counting the dots then cover it up with his or hand, visualizing (make a picture in your head) the dots. Then ask the child to uncover the dot pattern and call out how many. Also, have the student use materials such as glass gems or other same-shaped counters to build and represent the same quantity in many ways. For example, have the child count out three counters and then line them up, make a triangle, make an L, etc, counting after each change to ensure that there is conservation of quantity – that there is are still three! Students might like to create their own set of dot cards using circle stickers or bingo dabbers to take home to play games with.

What next?

1) If a student is having difficulty estimating, begin by using comparing language of more and less. Present two small sets of materials – *Which one has more? Which one has less?* Also, reduce the quantity of objects that a student is estimating and use different colours or spatial organization to support estimating. For example, in a set of 9, present the cubes as a set of five red cubes grouped together with four blue cubes.





2) If a student is fluent and confident when estimating quantities, apply this understanding to computational estimation and estimating when measuring (time, money, linear measurement, etc). Students could also analyze class sets of estimates or ranges, looking for patterns and representing the information in charts or graphs.

References

Common Core Math for Parents by Christopher Danielson
http://www.estimation180.com (website by Andrew Stadel)
K-7 Mathematics Curriculum, BC Ministry of Education, 2007
How Children Learn Number Concepts by Kathy Richardson
Developing Essential Understanding of Number & Numeration: Pre-K - Grade 2 *by National Council of Teachers of Mathematics (NCTM)*
How Many Marbles in the Jar? Estimation in the Early Grades by Jennifer Taylor-Cox, in *Teaching Children Mathematics*, December 2001, pages 208-214
K-2 Estimation Tasks powerpoint by Ryan Dent:
https://onedrive.live.com/view.aspx?resid=665A7678364788F3!146&ithint=file%2cpptx&app=PowerPoint&authkey=!AOBtsfYSUxZ1nUs

Children's Literature

Great Estimations – *Bruce Goldstone*
Greater Estimations – *Bruce Goldstone*
How Many Seeds in a Pumpkin? – *Margaret McNamara*
Counting on Frank – *Rod Clement*
Betcha! – *Stuart J. Murphy*

created for the BCAMT Reggio-Inspired Mathematics Inquiry Project/JN June 2016

Estimating

What is it?

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or determining the reasonableness of calculated values. Students need to know how, when and what strategy to use when estimating. Estimation is used to make mathematical judgments and develop useful, efficient strategies for dealing with situations in daily life (BC Ministry of Education, 2007).

Concepts and skills involved in estimating:

- estimate – to make a "smart guess," it is beyond just a random guess or "favourite number" and involves use of reasoning, comparing and strategy use
- reasonable – an important metacognitive process when estimating is for students to consider if their estimates are reasonable or not (this is something that young children may have difficulty with)
- range – an understanding that an estimate is not precise and that a reasonable range is important to consider and that size of the range depends on the context
- compare – estimation begins with comparing amounts (more, less, about the same as)
- referent – a referent is used as a comparative quantity and is sometimes called a benchmark or anchor when working with numbers; for example, when estimating a large quantity of beans, a small set of ten beans may be used as a visual referent to support estimating the large quantity

Why is it important?

Although we focus on estimating quantities here, there are many areas of mathematics in which students will apply estimation understanding and skills. Computational estimation and the concepts of underestimating, overestimating and "rounding" are all important as students begin to calculate with larger numbers. Estimation is also an important process when measuring – measuring time, money and length, for example. It is in measuring contexts as well as problem-based contexts involving calculations that the purpose of estimating is an important consideration. Sometimes it is better to overestimate, other times it is better to underestimate. This contextual purpose highlights the importance of developing the concept of a range of reasonableness with students – what is too high and what is too low, as made clear in Andrew Stadel's Estimation 180 tasks.

What to think about?

- Gather a variety of items to estimate.
- The size of the collection should vary. Begin with collections that might range from about 10 to up to 100.
- Materials should be of a consistent size and shape when estimating a quantity or set but make sure to vary the objects used – sometimes using small beads and other times using larger cubes. Encourage students to make connections about how the size of the object affects their estimation strategies.
- Vary estimation tasks – sometimes spread objects out on a mat for students to see and other times in a container. In both cases, it is important that students are not able to "count" the objects easily.
- Think about whether or not to count to find the "correct" amount after estimating. When you do count to compare actual versus estimate, this can help to develop students' estimation abilities but it also can be counter to the message of the importance of estimating.

Considerations when developing estimating tasks:

- purpose – What is the context, problem or purpose for estimating? When we ask students to estimate how many marbles in a jar, why would that be a worthwhile thing for them to do? What connections to themselves or to their world could you help them make to help them understand the purpose of practicing estimating?
- diverse experiences – What other experiences with estimating could be provided beyond estimating a set or a quantity of objects in a container? How might the language and ideas of estimating be connected across mathematical curricular content?
- range-based techniques – How do we limit students' understanding of the purpose of estimation if we always compare their estimates to a "right" answer? How might focusing on reasonable ranges support a more authentic understanding of estimating?
  (drawn from the work of Taylor-Cox, 2001)

What to do?

Begin with a mini-lesson using a small collection of objects. Spread the objects in front of you, having a piece of paper covering them. Explain to students that you are going to show them a set of objects and that you would like them to estimate how many there are. Remind them they do not need to try and count but should think of a strategy to help them estimate about how many there are. You might begin by contextualizing the task by asking, "Will there be enough for each of you to have one? More than enough? Not enough?" Uncover the set of objects for a few seconds and then cover them again. Have students share their estimates, their strategies and discuss what a reasonable range for the estimate would be. Have students consider when they might use estimation in their lives.

Build on one of the students' suggested strategies around comparing to something or using a referent by modeling this for students who may not have thought of this strategy. As a follow-up task, use an "estimation jar" filled with a set of objects with a referent nearby (1-20 objects depending on the size of the objects/quantity of the set). Have students share and compare their estimates and strategies.

What to look for?

Provide the students with both small and large collections or sets of materials both in containers and not (i.e. marbles, cubes, pinecones).

- Are students able to estimate reasonably?
- Do they look at or scan the quantity? Do you notice them visualizing?
- Are students able to explain their strategies? (ie. "I counted two cubes and then estimated based on how much space two took up.")
- If students count the quantity after estimating, do they make a connection to their estimate? Are they aware they have gone past or have not reached their estimate? Can they adjust their estimates based on new information or as you count the set?
- Do students adjust their estimates while counting or after hearing another student's reasoning? Are they making meaning of the situation?